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RADIATION OF THE INNER SURFACE OF A PERFORATED CIRCULAR CONE

S. N. Shevchenko and A. P. Makarov UDC 536.24

We analyze the results of a numerical solution obtained for the problem of finding special features of radiation and the presence of a perforation or channelling effect in anisotropic emission from the inner surface of a circular cone perforated with round holes.

The majority of engineering methods applied to calculate radiant heat exchange in assemblies and apparatus are based on the assumption that the radiation of the surfaces that constitute a part of a heat exchanging system is isotropic, i.e., it obeys the Lambert law, according to which the intensity of radiation of the surface is assumed to be constant along the direction. However, this condition does not hold for many actual surfaces. As indicated in [1, 2], the error occurring in the determination of the overall heat flux as a result of the assumption of isotropic radition can sometimes attain an appreciable value (25% and greater).

Perforated surfaces, even with the assumption of local diffuseness, have a very pronounced radiation anisotropy, thus allowing one to use them to carry out an effective control of radiation fluxes and heat exchange on the whole and optimize the radiative and thermal characteristics of objects. In works dealing with heat exchange in perforated systems an assumption is made concerning infinitely thin surfaces. And though even in this case the channelling or perforation effect manifests itself [3], the radiation anisotropy of an actual surface is not taken into account.

In what follows we consider the radiation of the inner surface of a circular cone with uniformly located perforations and the effect of optical and geometric characteristics and of the system on the whole on the net radiation flux.

Defining the angular coefficient as the ratio of the portion of the radiant flux incident on the surface irradiated or its element to the semispherical radiation of the irradiated surface (or its element), we have [1]

$$\varphi_{F_1,F_2} = \int\limits_{F_1}\int\limits_{F_2} \frac{\cos \alpha_1 \cos \alpha_2}{\pi S^2} f_1(\alpha_1) f_2(\alpha_2)\, dF_1 dF_2 \,. \tag{1}$$

The angular coefficient φ_{11} characterizes the radiation of the perforated cone inner surface onto itself. This is a determining angular coefficient, since, having calculated it, one can easily find the radiation flux from this surface into the space. Differentiating the side surface and transforming to cylindrical coordinates for an elemental area (Fig. 1) of the side surface, we obtain

$$dF = \frac{R\sqrt{R^2 + H^2}}{H^2} zdzd\vartheta \,. \tag{2}$$

Proceeding from the geometry of the system (Fig. 1), in cylindrical coordinates we have

$$S = \sqrt{\xi^2 \left[z_2^2 + z_1^2 - 2z_2 z_1 \cos(\vartheta_2 - \vartheta_1)\right] + (z_2 - z_1)^2} \,, \tag{3}$$

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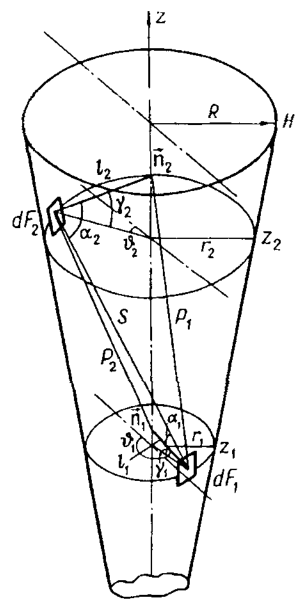


Fig. 1. Diagram for calculation of angular coefficient of radiation.

$$\cos\alpha_i = \frac{z_i^2\,\xi^2\,(1+\xi^2) + S^2 - [z_2 - z_1 + (3-2i)\,z_i\,\xi^2]^2 - z_i^2\,\xi^2}{2z_i\,\xi\sqrt{1+\xi^2}\,S}. \tag{4}$$

Taking into consideration that

$$f_i(\alpha_i) = 1 - 2\beta\left(\arccos\gamma_i - \gamma_i\sqrt{1-\gamma_i^2}\right)/\pi \quad \text{when} \quad \alpha_i < \alpha^*,$$

$$f_i(\alpha_i) = 1.0 \quad \text{when} \quad \alpha_i > \alpha^*, \tag{5}$$

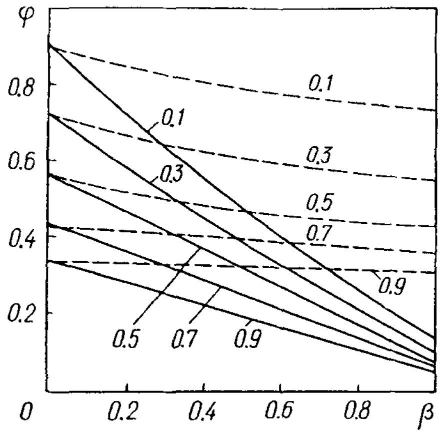
where $\alpha_i = \arctan(\mu^{-1})$, $\gamma_i = \mu \tan\alpha_i$, and substituting Eqs. (2)-(5) into Eq. (1), we obtain a 4-fold integral, which is not computed analytically.

To solve Eq. (1), we used the statistical-tests method. With 50,000 points, the integral converges to the third decimal place inclusive. Moreover, for calculation one can successfully use standard programs for the generation of pseudorandom numbers, even though some of them yield distributions that somewhat differ from a uniform one.

The results of calculation of the angular coefficient of radiation as a function of β at different values of ξ and at $\mu = 0.1$ and 1.0 are presented in Fig. 2. From the figure it is seen that as the perforation degree increases, the angular coefficient decreases, with this decrease being almost linear. A most substantial decrease occurs at small values of μ. The change in φ at large μ is insignificant. Thus, when the values of μ are large, the angular coefficient of radiation can be calculated with sufficient accuracy having transformed the well-known formula of Christiansen [4, 5] for two disks by letting one of the radii of the disks approach zero.

If we compare the obtained values of φ with the angular coefficient for a cone having an infinitely thin perforated surface, we see an appreciable difference in the dependence $\varphi = f(\beta)$ for the cases at $\beta = 1.0$. Evidently, in this case there are no interchannel surfaces. In a cone formed by an infinitely thin surface, the side surface area is equal to zero, i.e., the cone virtually does not exist. In this case $\varphi = 0$, i.e., there is no radiation from the system.

In a perforated cone whose surfaces have thickness this effect is not observed, because at $\beta = 1.0$ and $\mu \neq 0$ the surface area between perforations is equal to zero, but the area of the side surfaces of the channels differs from zero. In this case the perforated surface of the cone has, figuratively speaking, a honeycomb structure with the thickness of the walls of the cells tending to zero. It is evident that such a surface will emit also at $\beta = 1.0$, and the main contribution to radiation will be determined by the inner surface of the channels. When the values of μ



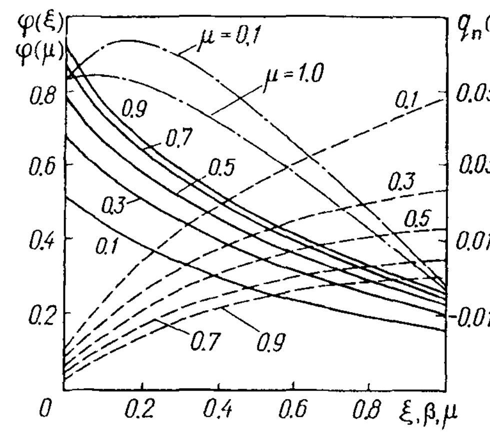


Fig. 2. Function $\varphi(\beta)$ at $\mu = 0.1$ (dashed curve), $\mu = 1.0$ (solid curve) at different values of ξ (at the curves).

Fig. 3. Function $\varphi(\xi)$ at $\beta = 0.6$ and different values of μ (solid curve); the function $\varphi(\mu)$ at $\beta = 0.9$ and different values of ξ (dashed curve), the function $q_n(\beta)$ at $\varepsilon = 0.1$, $\xi = 0.1$. μ is a parameter (dashed-dotted curve).

are small, the relative thickness of the wall of the cone decreases, which makes the function $\varphi(\beta)$ approach an analogous one for a cone with an infinitely thin surface, and the values coincide in the limiting case when $\mu \to 0$.

Figure 3 presents the function $\varphi(\mu)$ for $\beta = 0.9$ and different values of ξ. As μ increases (which is equivalent to a relative elongation of the perforation channels or thickening of the side surface of the cone), the total area of the emitting surface increases and, correspondingly, the value of φ is increased at a fixed β. This increase is typical for any values of μ. Here it should be noted that such a process of the change in φ is observed only at β = const. This can be explained by the fact that, though being connected with β through the size of a hole, the parameter μ exerts its influence on the change in the radiation of the surface inside of the perforation channels. When ξ = const, we have $\lim_{\mu \to 0} \varphi$ = const; so, for example, when $\xi = 0.1$, $\lim_{\mu \to 0} \varphi = 0.1$. It is evident that in this case one observes the case of an infinitely thin perforated surface [3]. Then the angular coefficient of radiation is a function only of β and ξ, and its calculation can be carried out by the formulas of [3]. This very figure also depicts the function $\varphi(\xi)$ at $\beta = 0.6$ for different values of μ. This dependence is nonlinear, and the values of φ decrease as ξ increases. We can see from the figure that φ is always smaller for a perforated cone than for a continuous one, and the thicker the wall of the cone, the closer the values of φ to $\varphi(\xi)$ for a continuous-surface cone.

For practical calculations of radiation in systems containing a perforated cone we obtained a formula that approximated the calculated data for the angular coefficient in the ranges of β, μ, and ξ of from 0 to 1:

$$\varphi = \varphi_0 - [(0.464\xi^2 - 0.794\xi - 0.736)\mu^2 - (1.05\xi^2 - 1.69\xi + $$

$$+ 1.49)\mu + 0.464\xi^2 - 1.14\xi + 1.0]\beta ,$$

where $\varphi_0 = 0.355\xi^2 - 1.075\xi + 1.01$ is the angular coefficient of the continuous cone. This formula determines φ with an error not exceeding 5%. Moreover, the error is decreased with a decrease in the parameters μ and ξ.

To determine the net radiation flux from the inner surface of a perforated cone the resolvent-zonal approximation [5] was used. The equation that determines this flux in dimensionless form can be simplified to the form

$$q_n = -\varepsilon\,[1 - \varphi(\xi, \mu, \beta)]\left[\frac{(1-\varepsilon)\,\varphi(\xi, \mu, \beta)}{1 - (1-\varepsilon)\,\varphi(\xi, \mu, \beta)} + 1\right](1 - \beta) .$$

The $q_n(\beta)$ dependences at $\mu = 0.1$ and 1.0, $\xi = 0.1$, and $\varepsilon = 0.1$ are presented in Fig. 3. It is seen from the figure that in the region with $0.1 < \beta < 0.3$ there is an inner-surface radiation maximum, with an increase in radiation occurring with a decrease in μ. The position of the maximum agrees well with the data of [3] for systems with infinitely thin perforated surfaces. With an increase in the thickness of the radiating surface, this effect decreases. Thus, we can conclude that the radiation that passes through the perforations of the system makes a greater contribution into the perforation effect than the radiation of the walls of the perforations, and in order to increase radiation from a system at fixed values of β and ξ, it is necessary to decrease the thickness of the surfaces that form the system.

The results obtained make it possible to represent more fully the mechanism of the channelling or perforation effect in modeling of an ideal black body. Thus, the analysis carried out makes it possible to explain easily the efficiency of a spherical and cylindrical model of an ideal black body with emission through a hole in a side surface compared to the widely used conical and cylindrical models with radiation through the base of the form.

NOTATION

dF_i, elemental area on surface of cone; R, radius of cone base; H, cone height; S, distance between elemental areas; r, z, and ϑ, cylindrical coordinates; $\xi = R/H$; h, surface thickness; d, diameter of perforations; $\mu = h/d$; β, degree of perforation; $f_i(\alpha_i)$, shape factor; α_i, angle between normal to dF_i and S; q_n, dimensionless net radiation flux; ε, integrated emissivity of surface; l_i, distance along normal from dF_i to z axis; p_i, distance between dF_i and the point of intersection of the z axis with the normal to the opposite elemental area. Subscripts: 1, 2, number of an elemental area.

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